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BRIDGING ENGINEERING MATHEMATICS AND CFD IN CONTEMPORARY FLUID MECHANICS EDUCATION

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Abstract. Contemporary fluid-mechanics textbooks increasingly introduce computational fluid dynamics (CFD), yet many engineering students reach advanced fluid mechanics without a connected understanding of vector fields, tensors, partial differential equations, matrices, and numerical discretization. This conceptual pedagogical study identifies the curricular gap between conventional engineering mathematics and the reasoning required for advanced fluid mechanics and CFD. Qualitative curriculum mapping traces the dependency chain from velocity fields to velocity-gradient and stress tensors, conservation laws, the Navier-Stokes equations, discretization, matrix systems, and verified numerical solutions. The paper then tests the constructive adequacy of a proposed one-semester course, Mathematical Foundations for Fluid Mechanics and CFD: every transition in the chain is assigned explicit content, an authentic task, and observable evidence of learning. This mapping supports the argument that the principal problem is not a weakness in modern fluid-mechanics textbooks, but the absence of a curricular bridge linking mathematical concepts to their physical and computational roles. The model is suitable for later empirical testing in engineering programs, including those in Uzbekistan.

Keywords: engineering mathematics, fluid mechanics education, tensor analysis, numerical modeling, CFD, curriculum integration, Navier-Stokes equations

Introduction

Engineering curricula are necessarily divided into courses, but knowledge itself does not follow those boundaries. Linear algebra may teach vectors, matrices, determinants, and eigenvalues; calculus may teach partial derivatives, gradients, and divergence; continuum mechanics may introduce tensors and stress; fluid mechanics may derive conservation equations; and numerical methods may present discretization and iterative solutions. CFD often appears later as either a specialized elective or the use of a commercial package. Each course can be internally logical while the overall sequence remains conceptually discontinuous. Students learn individual techniques without being shown clearly why one idea is needed for the next [1].

This fragmentation is more than a lack of prerequisites. A student may multiply matrices, differentiate vector functions, and reproduce the Navier-Stokes equations, yet still fail to

explain how a differential equation becomes a computer solution. The missing element is continuation from mathematical object to physical meaning, governing equation, and numerical representation. Engineering-education research has criticized curricula in which mathematical meaning is postponed, and students must reconnect separated subjects for themselves [2, 3].

Research aim and method

The study aims to define the missing connection and formulate an implementable response. It uses qualitative curriculum mapping to align concepts normally taught in engineering mathematics, continuum mechanics, fluid mechanics, numerical methods, and CFD. The proposed course is examined by a constructive adequacy test. First, the target competence is decomposed into necessary transitions from mathematical representation to trustworthy CFD interpretation. Second, each transition must be taught through explicit content, enacted in a task, and assessed through observable performance. A course that leaves any transition unaddressed reproduces the fragmentation it is intended to solve. This is a conceptual test, not evidence of measured achievement; empirical effectiveness must be established later through diagnostic assessments, interviews, and a CFD verification task [2, 3].

3. The mathematical and computational learning progression

The chain can be made visible through the example of fluid motion. For example, a vector represents velocity at one point. Then, assigning a vector to every point and time creates a velocity field. Differentiating that field produces the velocity-gradient tensor. Its symmetric part represents the local rate of deformation, whereas its antisymmetric part represents rigid-body rotation. Through a constitutive relation, deformation rate is connected to viscous stress. Divergence expresses the net outward flux from an infinitesimal region, while material acceleration combines local and convective change. These operations are not a collection of advanced symbols; together they provide the language required to express conservation of mass and momentum [4, 5].

$$\nabla \cdot u = 0 \quad (1)$$

$$\rho(\partial u / \partial t + u \cdot \nabla u) = -\nabla p + \mu \nabla^2 u + \rho g \quad (2)$$

At this point another curricular break often occurs. The governing equations are derived in fluid mechanics, but their analytical solutions are limited to simplified geometries and assumptions. Numerical modeling begins by dividing a continuous domain into nodes, elements, or control volumes and replacing derivatives and fluxes with algebraic approximations. The discretized equations for all locations are assembled into a coupled system. The matrix introduced earlier as an abstract rectangular array now becomes a representation of the differential operator, the mesh connectivity, the physical coefficients, and the imposed boundary conditions. The unknown vector contains values such as velocity, pressure, temperature, species concentration, or turbulent quantities [4].

$$Ax = b \quad (3)$$

This relationship explains why matrices are fundamental to numerical modeling rather than merely a preliminary mathematical topic. Sparsity occurs because each computational

location interacts mainly with nearby locations. Matrix conditioning influences the sensitivity of the computed solution. Eigenvalues relate to stability, wave behavior, and convergence. Iterative methods are required because realistic systems can contain millions of unknowns. When these meanings are not taught, students may remember matrix operations but fail to recognize the matrix structure inside a CFD solver.

The curricular claim can now be stated precisely. Introductory CFD understanding requires six connected transitions: point values to fields; fields to tensor measures; tensor measures to conservation equations; equations to a well-posed model; the continuous model to a discrete matrix system; and the computed system to verify physical evidence. These transitions are cumulative. For example, discretization has little meaning if the learner cannot interpret the governing terms, while a converged calculation is not trustworthy if the learner cannot test conservation, mesh dependence, and agreement with a benchmark. Table 1 shows that the proposed content covers every transition and makes it assessable.

Course content	Transition repaired	Authentic learning task	Evidence of learning
Vectors and fields	Point velocity to Eulerian field	Interpret and visualize $u(x,t)$; compute gradient and divergence	Explains spatial and temporal change in a flow field
Tensor foundations	Field derivatives to deformation and stress	Compute the velocity gradient; separate strain rate and rotation; infer viscous stress	Connects tensor components with observable deformation
Conservation laws	Local fluxes and forces to Navier-Stokes equations	Derive mass and momentum balances for one control volume and state assumptions	Explains the physical origin of every governing term
PDEs and boundaries	Governing equations to a well-posed model	Classify advection, diffusion, and pressure terms; justify initial and boundary conditions	Detects incompatible assumptions or boundary data
Discretization and matrices	Continuous model to algebraic system	Discretize a small diffusion-advection problem and assemble A , x , and b	Explains each coefficient through mesh, physics, and boundary conditions
Solvers and verification	Computed values to credible CFD evidence	Run a simple case; test residuals, conservation, mesh sensitivity, and a benchmark	Distinguishes numerical convergence from physical validity

Table 1. Constructive alignment of the proposed bridge course.

4. Consequences of the curricular gap

The most visible consequence is black-box modeling. Students may generate a mesh, select a model, and display contours without understanding the transformations between those steps. They may treat a converged residual as proof of physical correctness or report a plausible flow field without mesh assessment or validation. Fragmentation also prevents transfer: a learner may not recognize the gradient as a link between velocity variation and

strain rate, eigenvalues as indicators of stability, or a matrix as the assembled form of discretized equations. The knowledge may be present but stored in a form that is difficult to apply. Engineering-mathematics studies distinguish procedural performance from conceptual understanding [2, 3], while CFD education benefits when students inspect discretization and matrix equations rather than learn software operation alone [6, 7].

5. Contemporary fluid mechanics textbooks and the remaining gap

Contemporary textbooks have made important progress toward computational integration. Çengel and Cimbala's *Fluid Mechanics: Fundamentals and Applications* is a strong example: its sequence includes fluid kinematics, differential analysis, approximate Navier-Stokes solutions, and a dedicated introduction to CFD [4]. The remaining gap should not be attributed to such books. An undergraduate fluid-mechanics text cannot also provide full courses in tensor algebra, numerical analysis, sparse linear algebra, and programming. Specialized tensor texts themselves acknowledge the passage that must be made from linear algebra to continuum-mechanics tensor analysis [5]. The institutional curriculum must therefore supply the bridge that enables students to use the advanced textbook material.

6. Course content as a constructive proof

The proposed response is a one-semester bridge course entitled *Mathematical Foundations for Fluid Mechanics and CFD*, positioned after multivariable calculus and linear algebra and before, or concurrently with, advanced fluid mechanics and introductory CFD. Its content supplies constructive proof of curricular coherence. Modules 1 and 2 connect familiar calculus and linear algebra to kinematics and constitutive behavior. Modules 3 and 4 connect those meanings to conservation laws, modeling assumptions, and boundary conditions. Modules 5 and 6 connect the continuous model to matrix equations, solver behavior, and verification. Therefore, no conceptual handoff in the defined dependency chain is left implicit. This establishes structural sufficiency for repairing the specified gap, although it does not by itself establish the size of any learning gain.

The same canonical flow problem should be revisited in all six modules. Students first describe its velocity field, then calculate deformation and stress, derive its conservation equations, specify a well-posed model, assemble a small coefficient matrix, and finally evaluate a numerical solution. This repeated object makes continuation visible: each new representation is produced from the preceding one. Assessment follows the same logic. A student demonstrates integration only when he or she can translate in both directions among a physical flow, tensor notation, governing equations, matrix coefficients, and CFD output. Success should be judged with a common rubric covering mathematical correctness, physical interpretation, numerical reasoning, and verification, not by software operation alone.

For engineering programs in Uzbekistan, the course could first be introduced as an upper-level elective or interdisciplinary module shared by mathematics and engineering departments. A pilot should use matched pre- and post-course tasks: interpretation of a velocity-gradient tensor, derivation of a balance equation, assembly of a small matrix system, and verification of a CFD case. Rubric scores, student explanations, and comparison with a cohort following the conventional sequence would test whether the structural argument produces measurable gains before wider adoption.

7. Discussion and conclusion

The proposed course does not replace engineering mathematics, fluid mechanics, numerical methods, or CFD. Its contribution is a dependency-based model in which every required transition has corresponding content, practice, and evidence. The argument is therefore stronger than a list of desirable topics: omitting tensor foundations breaks the passage from fields to stress; omitting discretization breaks the passage from equations to matrices; and omitting verification leaves no basis for judging a CFD result. The six-module sequence is the minimum coherent chain defined by the target competence.

Knowledge fragmentation is therefore a significant obstacle to advanced fluid-mechanics and CFD understanding. Students need guided transitions from mathematical representation to continuum physics and from governing equations to numerical solutions. Modern textbooks such as Çengel and Cimbala provide an excellent fluid-mechanics framework and appropriately introduce CFD; the proposed bridge course supplies the connected tensor, matrix, and verification reasoning needed to engage fully with that material. Its structural adequacy is demonstrated by the complete content-task-evidence mapping, while its educational effectiveness remains a clear and testable proposition for a pilot study.

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